Web Unit Plan

Title: Fair Games

Description: Have you ever heard, "That's not fair" during a game? Any good game must be fair so that each player has an equal chance at winning. Students learn about the mathematics notion of fairness while participating in activities and games of chance. Students take on the role of game designer to create a new game for a toy company describing the rules for play and explaining why the game is fair. These new designers then present the game to an audience of invited guests.

At a Glance

Grade Level: 6-8

Subject sort (for Web site index): Mathematics

Subject: Mathematics

Topics: Probability and Statistics

Higher-Order Thinking Skills: Evaluation, Creativity, Problem Solving

Key Learnings: Degrees of Likelihood, Predicting Skills, Understanding Probability, Determining Fairness

Time Needed: Eight 45-minute lessons

Unit Summary

To learn about probability and fairness, students participate in several chance activities and examine a few games for fairness. Student groups become game designers who are asked to design a fair game for a toy company describing the rules for play and explaining mathematically why the game is fair. Finally, groups present their game to a fictional toy company's board of directors convincing them to sell their game.

Curriculum-Framing Questions

- **Essential Question**
  Is life fair?
- **Unit Questions**
  What is the likelihood that certain events will occur?
  What determines fairness?
- **Content Questions**
  What is probability?
  How do you measure the likelihood of an event?
  How do you determine and represent probable outcomes?
  What is the difference between experimental and theoretical probability?

Assessment Processes

View how a variety of student-centered assessments are used in the Fair Games Unit Plan. These assessments help students and teachers set goals; monitor student progress; provide feedback; assess thinking, processes, performances, and products; and reflect on learning throughout the learning cycle.

Instructional Procedures

Set the Stage

Ask students if they have ever been in a situation that was not fair. Pose the Essential Question, *Is life fair?* Break students into small groups and have them discuss the Essential Question and record their initial responses. Encourage them to talk about why they think life is fair or unfair, as well as what they mean by fair and what determines fairness. Ask several students to share their responses to the Essential Question and then tell them that they will begin a unit to learn how to use mathematics to determine the fairness of games.

Introduce a math journal to students. This journal will be used to record answers to questions, prompts, and problems.

What are the Chances? Activity

This activity addresses the Content Questions, *What is probability?* and *How do you measure the likelihood of an event?*

Overview of activity:
Introduce the idea of probability by discussing the likelihood of events occurring. Encourage students to focus on the language of probability as they use their life experiences to recall events that are certain, impossible, likely, and unlikely to happen. Record these events and introduce students to a probability scale, ranging from zero to one. This activity is intended to get students involved in talking about probability.

Materials needed:

- Trash can
- Ball of paper

Activity procedures:

1. Stand ten feet away from the trash can and hold the ball of paper in your hand. Ask the students, *What are the chances I will make it in the trash can on my first try?* Focus the discussion on vocabulary terms: certain, likely, unlikely, and impossible.
2. Ask students what the word “probability” means. Ask them to name situations that use probability.
3. Tell the class that probability can be expressed on a probability scale. Draw a number line on the chalkboard representing the scale. Ask the students to name a number that would best represent an event that is impossible (0 or 0%). Write “0 IMPOSSIBLE” at one end of the scale. Ask students to name events that are impossible such as: there will be 12 hours in the day tomorrow or, when your roll two dice you get a sum of 0. List student responses on chart paper to refer to throughout the unit.
4. Ask students to name a number that would best represent an event that is certain (1 or 100%). Encourage students to name events that are certain and record their responses in their math journals. For example: there will be 24 hours in the day tomorrow or, there will be seven days in the week next week.

5. Mark 1/2 or 50% on the scale and ask students what they think this means on a probability scale. Ask them to name events that would fall under "equally likely" events. Ask the students to make a prediction about the weather for tomorrow. Predict where on the probability scale best represents the likelihood of their weather prediction coming true. Students need to explain their reasons for predicting a particular place on the scale. If time allows, have students create their own graphic organizers in their math journals, making a probability scale and putting events at designated places along the scale.

Understanding Chance

The following are a series of activities that are meant to lead the students to understanding "chance".

What are the Chances? Mystery Pasta Activity

In preparation for this lesson, fill three bags with the following proportions of shell and elbow pasta shapes. Write the three populations on the chalkboard:

Bag 1: 8 shells, 16 elbow
Bag 2: 16 shells, 8 elbow
Bag 3: 4 shells, 20 elbow

- In this experiment students take turns reaching into the bag without looking, drawing out one pasta, and noting its shape. Explain to the class that the amounts of shell and elbow pasta in each bag is written on the board but the bags are not labeled so they don't know which bag has which population of pasta. Select a student to choose one of the bags and tell the class that the student's task will be to try and figure out which population of pasta is in the bag without looking inside, but instead by using some mathematical ideas of probability. Then the student will replace the pasta and someone else will have a turn.

- Pose the Content Question, *How do you determine and represent probable outcomes?* Lead a discussion to generate answers to this question. Write the numbers one through six in a column on the chalkboard and explain to the class that they will need to keep track of the results. Begin the experiment, recording the results and shaking the bag each time a student has a turn. After six tries, ask the class, *What does the information from our sample tell you about what's in the bag*? After a few guesses, do six more tries (numbering 7-12), and recording the results. Ask students, *What fraction of our sample came up shells?*

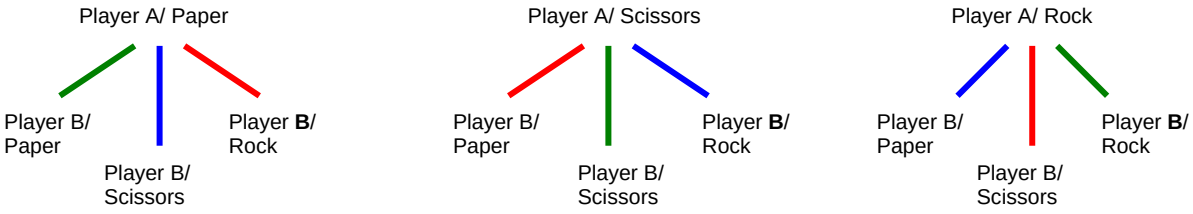
- Have students work in groups to figure out what fraction each of the populations A, B, and C is made up of shells. Have them compare this to the sample and predict which of the populations they think it is and why. After discussing their ideas and rationale, tell students that mathematicians have experimented and found that when you make many random draws as they are with the pasta, a pattern emerges. Probability is a way to predict that pattern. Return to the activity and continue drawing until most students can see a pattern emerging and are able to confidently predict what is in the bag. Then empty the bag to check if their prediction is true.

- Individual assessment in math journals: Use the students' response to the following problem to gain insights into what students learned and what misconceptions they may have:

    - *There is some pasta in a bag. Students took turns drawing out pasta noting its shape and replacing it. After 12 draws, they had drawn 6 shells, 4 elbows and 2 bow-ties. Write what you know for sure about what is in the bag and what you know is probably true.*

Is it a Fair Game? Rock, Paper, Scissors

Officially known as Rock Paper Scissors or RPS, this game is also known in parts of the world as Jenken, Jan Ken Pon, Roshambo, Shnik Shnak Shnuk, Ching Chong Chow, Farggling, Scissors Paper Stone. Divide the class into pairs (player A and player B) and have them play the game 15 times. Use chart paper or an overhead projector to record the results of player A in red and player B in a different color (How many A players won game 1,2,3,etc? How many B players won? How many ties?) Compare the results. Ask the class, *Is this game fair?* (explain that this means equal chance of winning for all players) Ask students to explain why they think it is fair. Try to elicit from students that it is fair because each student has an equally likely or equal chance of winning (50% or 1/2). Introduce students to a tree diagram as a visual tool for keeping track of the possible outcomes of this game: This is known as a probability tree. To address the Content Question, *What is the difference between experimental and theoretical probability?* compare this mathematical model with what happened when students played the game (theoretical vs. experimental probability).



Player A wins 3/9 or 1/3
Player B wins 3/9 or 1/3
Tie 3/9 or 1/3

Ask the students to play the game now with three players using the following rules:

- A wins if all three hands are the same
- B wins if all three hands are different
- C wins if two hands are the same

Ask students to consider the following questions: *Is this game fair? Why or why not? What determines fairness?* Ask students to construct a probability tree in their math journals to determine the possible outcomes (There will be 27 outcomes—three more branches off of each of the above nine possibilities. It is not fair because player C has more chances of winning than players A and B)

Remind students of the Essential Question they discussed at the beginning of the unit, *Is life fair? Does fairness in life relate to fairness in games and if so, how? And if not, why not?*

Rolling Dice: What are the Chances? Activity

Introduce the activity by discussing the possible outcomes that can be obtained when a die is rolled. Students should be able to identify that the possible outcomes are the numbers from 1 to 6. Then ask, *What are the possible sums if the two dice are rolled?* Have students work in groups to investigate the chances for rolling a particular sum. Have each person in the group create a number line for the possible sums (2,3,4,5,6,7,8,9,10,11,12) and place "x's" each time the sum is rolled. Have students roll the dice 15 times. Create a classroom frequency distribution graph (a number line with the "x's" to represent how many times each sum occurred). Ask students to compare their own group data to the whole class data. Ask students, *Are all sums equally likely to occur? If not, which ones are more likely to occur and which ones are least likely to occur?*

Introduce students to the idea that a table can be a useful tool in showing the possible outcomes (mathematically) of the sums of two dice. After getting students started on the table in their math journals, have them complete it:

		First Die					
		1	2	3	4	5	6
Second Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Ask students the following questions:
Which sum is most likely to occur on the next roll of dice? Least likely? Why?
How many total possible outcomes? (36)
How many times does each sum appear in the table?
What does this tell us? (The probability of that sum occurring; for example: 9 appears 4 times, so there is 4/36 or 1/9 probability of rolling a 9). Have them record responses in their math journals.

Looking at the Competition

In the following activity, students create a fair game based upon what they've learned in the previous activities and game. Bring what they've learned together by providing several games that use probability and chance. Allow students to play the games while recording why or why not they think the game is fair. Once student groups have played at least two different games, have the whole group discuss and list the common reasons the games were fair and how chance was involved.

In math journals, have students reflect on what they've learned from playing the games and brainstorm ideas for designing their new game. Offer a list of questions for to students to think about.

- *What determines fairness?*
- *What makes a game fun?*
- *How can probability be used?*
- *What are some rules that can be used?*

Putting it all Together

Have students share with their group in round-robin fashion what they brainstormed in their journals the previous day. Students then apply what they have learned as they take on the roles of game designers responding to an advertisement of a toy company that wants to create and sell a new game for children ages 11-13. Create an environment that fosters creative thinking by having students give and receive peer feedback and invite local business owners to share in the process of creating a product to sell. Each team of designers needs to create a game using number cubes, cards, or pasta to advance play, describe the rules for play, and explain why the game is fair using probability and graphical organizers (tables, lists, tree diagrams). Have students refer back to their math journals to connect what they've learned to create a new game. Have them create a multimedia presentation of their game to present to the fictional board of directors (parents, school faculty, local toy and business representatives) and address the Curriculum-Framing Questions. Show the sample presentation to students as an example and give students opportunities to ask questions and get any clarification needed. Hand out the project rubric and presentation checklist to discuss project expectations. Have students use the checklist to guide the creation of the slideshow presentation. Check for student understanding and guide students in using the rubric and checklist to create quality work.

To help students with the planning and implementing of their game idea, encourage students to use the following guiding questions to promote metacognition skills:

- *What information do I need?*
- *What resources do I have?*
- *What are the smaller tasks within this big project?*
- *What do I have to do in a particular order and what can I do any time?*
- *What problems might come up and how do I handle them?*

Model a think-aloud beforehand, so students are aware of strategies to use while exploring these questions in-depth. While students are using these metacognitive guiding questions, take anecdotal notes to document students' thinking processes.

Game Night

Invite parents, school faculty, and local toy and business representatives to attend a Game Night to recognize student work and learning. Students present their slideshows to the participants and then have time to play the games. Guests are invited to give students feedback about their game.

Wrapping Up

Return to the Essential Question, *Is life fair?* Ask students to think about how they responded to the question at the beginning of the unit. Have them write their thoughts in their journals about fairness, chance, and probability. Encourage them to write about what they have learned about these things over the course of the unit and to provide as much detail and examples as possible. As a final assessment, students fill out the self-reflection to reflect on what they've learned.

Prerequisite Skills

- Compare and operate with fractions, decimals, and percents
- Use ratio and proportions in problem solving

- Use graphs to organize, display, and interpret data
- Familiarity with multimedia presentation software

Differentiated Instruction

Resource Student

- Make modifications as dictated in the student's IEP.
- Provide visual aids and examples (documents, photos, and examples from this Unit Plan can be helpful).
- Supply an outline of the tasks and timeline for the project (including milestones).
- Select group best suited to work with this student.
- Provide extra time as needed to complete individual assignments.
- Students can bring in their favorite games and analyze whether they are chance or strategy games, providing a justification for their analysis

Gifted Student

- Students can bring to class games of their own and explain how they demonstrate the concept of fair games.
- Have the students consider whether it is a game of strategy or chance and explain their reasoning.
- Have students analyze the game for fairness and probability.
- Students can investigate games from different cultures and analyze them for fairness and probability, strategy, or chance.
- Students can write to toy companies who create board games. Addresses can be found in a library or Web site. Students can inquire about how companies think up new games, the guidelines they use to design them, and the marketing concerns they need to consider. They could then write a report or create a presentation to teach the class what they have learned.
- Students can go to a toy store that has a variety of different types of games and discuss some of the games with a knowledgeable representative. For instance, which games are games of chance and which are games of strategy? They could compile a list of questions to ask the representative and write a report or create a presentation to teach the class what they have learned.

English Language Learner

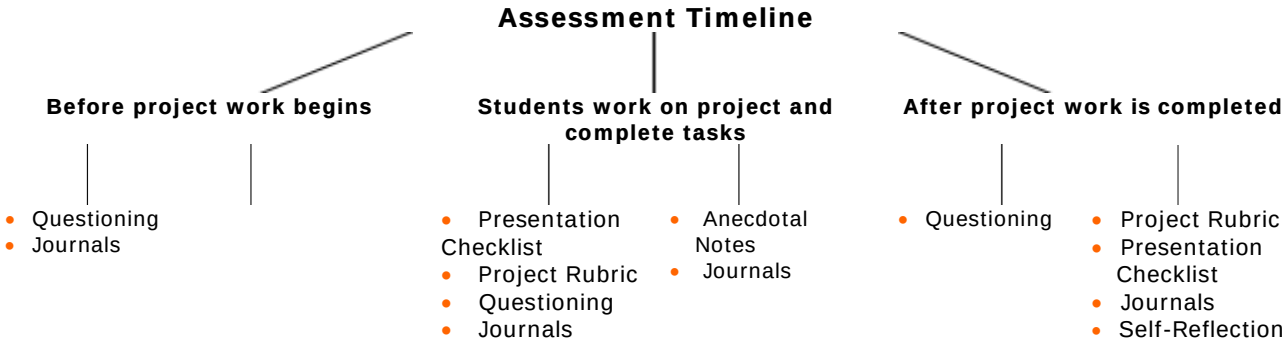
- Provide visual aids and examples (documents, visuals, and examples from this Unit Plan can be helpful).
- Try to use example games from student's native culture in an effort to link the unit to student's prior knowledge and experiences.
- Utilize the hands-on work of paper, scissors, rock, drawing pasta and rolling dice as well as the visual organizers (frequency graph, tables, visual reasoning) in helping students understand and conceptualize the content.

Credits

A teacher participated in the Intel® Teach Program, which resulted in this idea for a classroom project. A team of teachers expanded the plan into the example you see here.

THINGS YOU NEED (highlight box)

Assessment Plan



Quality of journal entries helps both teacher and students to monitor progress and understanding of content. The presentation checklist guides student learning, keeps them on track, and provides a self-assessment for progress. Questioning strategies throughout the unit allow for students to develop higher-order thinking skills and process content. A project rubric is introduced and assesses student learning and guides student project work. Anecdotal notes document student learning and guide teaching. To culminate the unit students complete a self-reflection.

Targeted Content Standards and Benchmarks

Targeted NCTM Content Standards

Probability and Statistics Standard for Grades 6-8
In grades 6-8 all students should:

- Understand and apply basic concepts of probability.
- Understand and use appropriate terminology to describe complementary and mutually exclusive events.
- Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
- Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

Problem-Solving Standard for Grades 6-8
In grades 6-8 instructional programs should enable all students to:

- Solve problems that arise in mathematics and in other contexts.
- Build new mathematical knowledge through problem-solving.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem-solving.

Connections Standard for Grades 6-8
In grades 6-8 instructional programs should enable all students to:

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Student Objectives

Students will be able to:

- Expand meanings of fractions to include probability as an estimate of expected results over a long period
- Recognize whether events are equally likely
- Know what probability is and define its properties:
  - Probability is a number between 0 and 1
  - Sum of probabilities of all outcomes is 1
  - Probability of 0 indicates an event that never happens and probability of 1 indicates an event that always happens
- Collect, organize, and analyze data to investigate events involving chance
- Use experiments to find the relative frequency of an event and use findings to predict behavior over time
- Recognize that larger number of trials produce better estimates of relative frequencies of events
- Use organized lists, tree diagrams, and tables to investigate probabilities
- Find probabilities for randomly pulling items from a bag
- Understand that in a fair game, each player has an equal probability of winning
- Decide whether a game is fair using relative frequencies and analyses

Materials and Resources

Supplies

- A variety of strategy and chance games (Any card or dice game such as: Yahtzee* and UNO*)
- Two types of pasta (two colors or two shapes)
- Paper bags
- Dice (two per group)

Internet Resources

The Shoder Foundation Web site offers technological tools for probability experiments and analyses. This site offers computer simulated probability experiments with pre-set spinners, make your own spinners, and dice sums allowing students to experiment with large numbers of spins or rolls and analyze the frequency table of results. This is a great site for exploring and learning that a large number of trials produce better estimates of relative frequencies of events. Below are direct links to specific activities:

A: www.shodor.org/interactivate/activities/prob/index.html*
Pre-Set Spinners, Make-Your-Own Spinners and Dice Sums with frequency tables

B: www.shodor.org/interactivate/activities/race/index.html*
Racing Game with one die

C: www.shodor.org/interactivate/activities/racing/index.html*
Racing Game with two dice

D: www.shodor.org/interactivate/activities/dice/index.html*
Two dice and a table

E: www.shodor.org/interactivate/activities/spinner/index.html*
Adjustable spinners

F: www.shodor.org/interactivate/activities/chances/index.html*
Crazy Choices activity allows the user to run up to three different games of chance at once, allowing for comparisons of experimental and theoretical probabilities.

Other sites that offer probability software and games include:

- The National Library of Virtual Manipulatives
  www.matti.usu.edu/nlvm/nav/category_g_1_t_1.html*
  This site is the National Library of Virtual Manipulatives and includes spinners that students can create and spin for results.
- Chances
  http://nces.ed.gov/nceskids/chances/*
  This site is the National Center for Education Statistics and involves rolling dice virtually. Students can choose how many times to roll two dice and the results are shown graphically as well as in a list form for die 1 and die 2.
- Scholastic – 4 Great Math Games
  http://teacher.scholastic.com/lessonrepro/lessonplans/grmagam.htm*
  Favorite activities and games based on probability from Marilyn Burns

Technology - Hardware

- Computer to complete slideshow presentations, and view Web sites
- Projector to show student examples and model expectations
- Internet connectivity to browse the Web sites

Technology - Software

- Multimedia software to complete student slideshow presentations
- Word processing software to complete teacher support documents
- Internet browser to browse the Web sites